

BINARY CODED DECIMAL ADDITION

Field of the invention

The present invention relates to a binary coded decimal (BCD) addition, including an
5 adder circuit.

Background

In BCD arithmetic, at least four binary bits are needed to represent the numbers from 0 to
9. There are sixteen possible bit combinations using four binary bits, but only ten (i.e., 0
10 to 9) are valid BCD digits. Traditionally, BCD adder circuits adjust the binary sum after
the addition has been completed if the value is more than nine (i.e., 1001_2). For example,
whenever the unadjusted sum of two BCD digits produced a carry out (i.e., when the sum
exceeds 1001_2), the sum is corrected by adding 0110_2 .

A known BCD adder, described in "Logic and Computer Design Fundamentals", M
15 Morris Mano and Charles R Kime, Pearson Educational Asia, second edition, 2001 is
shown in **Fig. 1**. The BCD adder circuit **5** uses a four-bit binary adder **6** to add two BCD
operands **A(3)** to **A(0)** and **B(3)** to **B(0)**, and an input carry C_{in} to produce an intermediate
sum **Z(3)** to **Z(0)**. The adder circuit **6** also includes two levels of logic gates **7, 8, 9** to
calculate the BCD carry output **Cout** from the intermediate sum. If there is a BCD carry,
20 a correction factor of 0110_2 is added with the intermediate sum using a further binary
adder **10**.

The BCD adder circuit **5** shown in **Fig. 1** thus uses 10 levels of gates to perform a 4-bit
addition. For a 16-bit adder circuit, the corresponding binary adder circuit requires 16
levels of gates. However the critical path is the generation of the carry **Cout**, and gate
25 levels required to implement this is 22 for 16-bit BCD addition.

Another BCD adder circuit is taught in US Patent No. 4,805,131 (Adlietta et al, assigned
to Digital Equipment Corporation), issued on February 14, 1989. The BCD adder of
Adlietta et al has three stages. In the first stage, 0110_2 is added with the input BCD
operands without considering the input carry. In the second stage the sum and carry
30 vectors are added using a carry look ahead network to generate final carry and propagate
vectors. In the third stage a correction circuit adds 1010_2 if needed.

The logic terms in the calculation of a FINAL_CARRY vector increases as the number of bits of the BCD operands increases. Also, the carry vector calculated is propagated through every bit position. The bit terms of the FINAL_CARRY vector must propagate through every bit position. As the bit position increases from least significant bit position 0 through most significant bit position 15, the number of logical terms required to produce the terms of the FINAL_CARRY vector also increases. Therefore, more sophisticated logic gates are required to produce the more significant bit terms and keep the number of logic levels and associated delay to a minimum.

For 16-bit BCD addition the BCD adder circuit of Adlietta et al uses only 12 gate levels compared to 22 gate levels in the BCD adder circuit of Fig. 1. But the logic terms required for the implementation of the FINAL_CARRY of BCD adder of Adlietta et al is much higher. Also, when the number of bits of the BCD operands increase, the logic terms for calculating the FINAL_CARRY also increases, which makes the implementation complex for more than 16-bits. There also is no reusability, meaning extending similar structures when the number of bits of the input BCD operands increases.

There remains a need for BCD addition circuits with lesser a number of gate levels as well as lesser number of logic terms without requiring any sophisticated logic gates. There is also a need to increase the reusability of logic gates in such adders.

Summary

The binary coded decimal (BCD) adder circuit adds two BCD encoded operands, with an input carry bit, and produces a BCD encoded sum. The adder includes three stages. The first stage receives two BCD encoded operands as inputs, groups the inputs into contiguous blocks of 4-bits each, computes an intermediate sum vector and carry vector without considering the input carry bit, and also computes propagation and generate functions for each 4-bit group. The second stage is a carry look ahead circuit which computes all carries from the input carry, and the propagate and generate functions of the 4-bit groups from the first stage. The third stage adjusts the intermediate sum vector with, pre-correction factors which depend upon the input carry and the carries generated from the second stage and the carry vectors from the first stage.

This invention reduces the time required to perform a BCD addition by reducing in the number of gate levels of the critical path for adding two BCD operands of 16-bits each with an input carry to 12. Only 15 gate levels of delay arise for adding two 64-bit BCD encoded operands with an input carry. The gate levels thus increase only by three, from
5 twelve to fifteen, when the input BCD operands bit size increases from sixteen to sixty-four.

Since the BCD addition is carried out with 4-bit groups, the same logic structure can be extended to provide a 16-, 32- or 64-bit BCD adder with extra levels of carry look ahead circuit, which increases the reusability of the logic.

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Description of drawings

Fig. 1 shows a known BCD adder circuit.

Fig. 2 shows a 16-bit BCD adder circuit embodying the invention.

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Fig. 3 shows detail of the circuit of **Fig. 2** when an addition of a first group of bits is performed.

Fig. 4 shows a circuit for calculating first intermediate sum and carry vectors.

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Fig. 5 shows a circuit for calculating second intermediate sum and carry vectors.

Fig. 6 shows a circuit for calculating third intermediate sum and carry vectors.

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Fig. 7 shows a carry look ahead circuit.

Fig. 8 shows a circuit for performing a SUM(0) correction.

Fig. 9 shows a circuit for performing a SUM(1) correction.

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Fig. 10 shows a circuit for performing a SUM(2) correction.

Fig. 11 shows a circuit for performing a SUM(3) correction.

Fig. 12 shows a stage 1 circuit for a 64-bit BCD adder circuit.

Fig. 13 shows a carry look ahead circuit for the 64-bit BCD adder circuit.

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Fig. 14 shows a sum correction circuit for the 64-bit BCD adder circuit.

Detailed description

Overview

- 10 The BCD addition of two operands each of n bits with an input carry bit will be described with reference to **Fig. 2**. A BCD adder circuit **12** generally has three stages, and the number of bits of the operands is $n=16$. The addition is performed after grouping the n -bits into contiguous (or successive) groups of 4-bits (i.e., bits 0-3, bits 4-7, bits 8-11, bits 12-15).
- 15 The first stage, represented by four circuits **13, 14, 15, 16**, generates intermediate sum and carry vectors and propagate and generate functions. The second stage has a carry look ahead circuit **17** to generate carries from the input carry, and the propagate and generate functions from the first stage. The third stage includes four circuits **18, 19, 20, 21** that generate a final sum from the intermediate sum vectors based on pre-correction factors
- 20 which depend on the input carry and the carries generated from carry look ahead circuit **17** and first stage circuits **13, 14, 15, 16**.

First stage

Consider the 16-bit BCD addition of two BCD encoded operands **A(15) to A(0)** and **B(15) to B(0)**.

- 25 The addition of first group of 4-bits **A(3) to A(0)** and **B(3) to B(0)** by the circuit **13** is explained with reference to **Figs. 3, 4, 5** and **6**. The circuits **14, 15, 16** for the other three groups of 4-bit addition logic are of the same form.

As shown in **Fig. 3**, three intermediate sum vectors **FS[3:0]**, **SS[3:0]**, and **TS[3:0]**, (where “F” = first, “S” = second and “T” = third) and carry vectors **FC[4:0]**, **SC[3:0]**,

- 30 **TC3 to TC0** are calculated by sequential generator circuits **22, 23, 24**. The third sum and

carry vectors **TS[3:0]** and **TC3 to TC0** are provided to a respective sum correction circuit 18 of the third stage.

The circuit 22 computes the first intermediate sum vector **FS(3) to FS(0)** and carry vector **FC(4) to FC(1)** by Half-Adders (XOR and AND gates) as shown in Fig. 4. The logical

5 operations performed are:

$$\mathbf{FS(3) = A(3) \text{ XOR } B(3)} \quad (\text{gate 37})$$

$$\mathbf{FS(2) = A(2) \text{ XOR } B(2)} \quad (\text{gate 35})$$

$$\mathbf{FS(1) = A(1) \text{ XOR } B(1)} \quad (\text{gate 33})$$

$$\mathbf{FS(0) = A(0) \text{ XOR } B(0)} \quad (\text{gate 31})$$

10
$$\mathbf{FC(4) = A(3) \text{ AND } B(3)} \quad (\text{gate 38})$$

$$\mathbf{FC(3) = A(2) \text{ AND } B(2)} \quad (\text{gate 36})$$

$$\mathbf{FC(2) = A(1) \text{ AND } B(1)} \quad (\text{gate 34})$$

$$\mathbf{FC(1) = A(0) \text{ AND } B(0)} \quad (\text{gate 32})$$

$$\mathbf{FC(0) = '0'}$$

15 The second intermediate sum vector of the first stage **SS(3) to SS(0)** and carry vector **SC(3) to SC(2)** are calculated by the circuit 23, including Half-Adders, as shown in detail in Fig. 5. The logical operations performed are:

$$\mathbf{SS(3) = FS(3) \text{ XOR } FC(3)} \quad (\text{gate 43})$$

$$\mathbf{SS(2) = FS(2) \text{ XOR } FC(2)} \quad (\text{gate 41})$$

20
$$\mathbf{SS(1) = FS(1) \text{ XOR } FC(1)} \quad (\text{gate 39})$$

$$\mathbf{SS(0) = FS(0) \text{ XOR } FC(0) = FS(0); \text{ since } FC(0) = '0'}$$

$$\mathbf{SC(3) = FS(2) \text{ AND } FC(2)} \quad (\text{gate 42})$$

$$\mathbf{SC(2) = FS(1) \text{ AND } FC(1)} \quad (\text{gate 40})$$

$$\mathbf{SC(1) = FS(0) \text{ AND } '0' = '0' ; \quad \text{since } \mathbf{FC(0) = '0'}$$

$$\mathbf{SC(0) = '0'}$$

The third intermediate sum vectors of the first stage **TS(3)** to **TS(0)** and a carry **TC3** are calculated by the circuit **24**, including Half-Adders, as shown in detail in **Fig. 6**. The

5 logical operations performed are:

$$\mathbf{TS(3) = SS(3) \text{ XOR } SC(3) \quad \quad \quad (\text{gate } 46)}$$

$$\mathbf{TS(2) = SS(2) \text{ XOR } SC(2) \quad \quad \quad (\text{gate } 44)}$$

$$\mathbf{TS(1) = SS(1) \text{ XOR } SC(1) = SS(1); \quad \text{since } \mathbf{SC(1) = '0'}$$

$$\mathbf{TS(0) = SS(0) \text{ XOR } SC(0) = FS(0); \quad \text{since } \mathbf{SC(0) = '0' \text{ and } SS(0) = FS(0)}$$

10
$$\mathbf{TC3 = SS(2) \text{ AND } SC(2) \quad \quad \quad (\text{gate } 45)}$$

$$\mathbf{TC2 = SS(1) \text{ AND } SC(1) = '0' ; \quad \text{since } \mathbf{SC(1) = '0'}$$

$$\mathbf{TC1 = SS(0) \text{ AND } SC(0) = '0' ; \quad \text{since } \mathbf{SC(0) = '0'}$$

$$\mathbf{TC0 = '0'}$$

15 Comparing **Figs. 4-6**, it can be seen that the design has been optimized such that Half-Adder circuits reduce to XOR gates or even to being straight through in some instances.

The propagate function **P3-0** and the generate function **G3-0** for the first group of 4-bits are calculated from the intermediate sum and carry vectors, by the combination logic shown in **Fig. 3** represented by gates **25, 26, 27, 28, 29, 30**. The logical operations performed are:

20
$$\mathbf{P3-0 = TS(3) \text{ AND } TS(0) \quad \quad \quad (\text{gate } 30)}$$

$$\mathbf{G3-0 = (SS(1) \text{ AND } (SS(3) \text{ OR } SC(3))) \text{ OR } (SS(3) \text{ AND } (SS(2) \text{ OR } SC(2))) \text{ OR } FC(4)}$$

(gates **25, 26, 27, 28, 29**)

25 Thus the functions **P3-0** and **G3-0**, along with other 4-bit group propagate and generate functions **P7-4, P11-8, P15-12, G7-4, G11-8** and **G15-12**, are inputs to the second stage carry look ahead circuit **17**.

Second stage

The second stage carry look ahead circuit 17, calculates the carry values of all groups of 4-bits from the propagate and generate values of the first stage and the input carry bit **C0** as shown in **Fig. 7**.

5 C4 = G3-0 OR (P3-0 AND C0) (gates 47 and 57)

C8 = G7-4 OR (P7-4 AND G3-0) OR (P7-4 AND P3-0 AND C0) (gates 48, 49 and/or gate 58)

C12 = G11-8 OR (P11-8 AND G7-4) OR (P11-8 AND P7-4 AND G3-0) OR (P11-8 AND P7-4 AND P3-0 AND C0) (gates 50, 51, 52 and 59)

10 C16 = G15-12 OR (P15-12 AND G11-8) OR (P15-12 AND P11-8 AND G7-4) OR
(P15-12 AND P11-8 AND P7-4 AND G3-0) OR (P15-12 AND P11-8 AND P7-4
AND P3-0 AND C0) (gates 53, 54, 55, 56 and 60)

These carries are provided to the third stage.

The carry **C16** will be calculated only for a 16-bit BCD addition. If 32-bit or 64-bit BCD addition is required, then propagate function **P15-0** and generate function **G15-0** will be produced instead of **C16** to be provided as an input to the 64-bit carry look ahead network, shown in **Fig. 13**. The propagate and generate functions **P15-0** and **G15-0** are calculated as:

P15-0 = P15-12 AND P11-8 AND P7-4 AND P3-0

20 **G15-0 = G15-12 OR (P15-12 AND G11-8) OR (P15-12 AND P11-8 AND G7-4)**
OR (P15-12 AND P11-8 AND P7-4 AND G3-0)

The values **P3-0, P7-4, P11-8** and **P15-12** are propagate functions and **G3-0, G7-4, G11-8** and **G15-12** are generate functions from the first stage circuits 13,14,15 and 16 of Fig. 2.

25 Third stage

The third stage circuits **18, 19, 20, 21** adjust the sum with pre-correction factors which depend on the carries generated from the first stage circuits **13, 14, 15, 16**, the second stage carry look ahead circuit **17** and, in one case, the input carry **C0**.

- 5 In general, the sum correction for a 4-bit block(say N down to N-3) adds first stage 4-bit sum vector **TS[N:N-3]** and a pre-correction factor. The pre-correction factor is one of the values of $(TC_N000)_2$ or $(TC_N001)_2$ or $(TC_N110)_2$ or $(TC_N111)_2$ which will be decided based on the carry C_{N-3} of previous 4-bits(N-4 down to N-7) and the carry C_{N+1} of current 4 bits(N down to N-3).

For example, the general steps for sum correction block of bits 7 to 4 (where N = 7) are:

10 Step 1

One of the values "TC₇ 0 0 0", "TC₇ 0 0 1", "TC₇ 1 1 0", "TC₇ 1 1 1" will be selected by C₈ (Carry for bits 7 to 4) and C₄ (carry of previous 4 bits i.e. bits 3 to 0).

- if C₈=0 and C₄=0 => selection of TC₇000. (TC₇ is a single bit having value '1' or '0')
- 15 if C₈=0 and C₄=1 => selection of TC₇001
- if C₈=1 and C₄=0 => selection of TC₇110
- if C₈=1 and C₄=1 => selection of TC₇111

Step 2

The above selected value will be added with first stage SUM vector **TS[N:N-3]**.

20

The first of the 4-bit group sum correction circuits, namely circuit **18**, will now be explained in detail.

- The inputs to the sum correction circuit **18** are the third intermediate sum vectors **TS(3)** to **TS(0)**, the intermediate carry **TC3** from the first stage the input carry **C0** and the carry **C4**
- 25 from the second stage carry look ahead circuit **17**.

If the Pre-correction Factor (PF) is taken as **PF(3)** to **PF(0)**, and the intermediate sum is **TS(3)** to **TS(0)**, then the Final Sum will be obtained from the following steps, which have four levels of calculations.

In the first level, a First Correction Sum vector **FCS** and a First Correction Carry vector **FCC** are calculated from the PF vector (**PF(3)** to **PF(0)**) and the TS vector (**TS(3)** to **TS(0)**) as shown in **Table 1**.

5	First_Correction_Sum	First_Correction_Carry
	-----	-----
	FCS(3) = TS(3) XOR PF(3)	FCC(3) = TS(2) AND PF(2)
	FCS(2) = TS(2) XOR PF(2)	FCC(2) = TS(1) AND PF(1)
	FCS(1) = TS(1) XOR PF(1)	FCC(1) = TS(0) AND PF(0)
10	FCS(0) = TS(0) XOR PF(0)	FCC(0) = '0'

TABLE 1

In the second level, a Second Correction Sum **SCS** and a Second Correction Carry **SCC** are calculated from first level outputs **FCS(3)** to **FCS(0)** and **FCC(3)** to **FCC(0)** as shown in **Table 2**.

	Second_Correction_Sum	Second_Correction_Carry
	-----	-----
	SCS(3) = FCS(3) XOR FCC(3)	SCC(3) = FCS(2) AND FCC(2)
20	SCS(2) = FCS(2) XOR FCC(2)	SCC(2) = FCS(1) AND FCC(1)
	SCS(1) = FCS(1) XOR FCC(1)	SCC(1) = FCS(0) AND FCC(0)
	SCS(0) = FCS(0) XOR FCC(0)	SCC(0) = '0'

TABLE 2

In the third level, a Third Correction Sum **TCS** and Third Correction Carry **TCC** are calculated from second level outputs **SCS(3)** to **SCS(0)** and **SCC(3)** to **SCC(0)** as shown in **Table 3**.

5	Third_Correction_Sum	Third_Correction_Carry
	-----	-----
	TCS(3) = SCS(3) XOR SCC(3)	TCC(3) = SCS(2) AND SCC(2)
	TCS(2) = SCS(2) XOR SCC(2)	TCC(2) = SCS(1) AND SCC(1)
	TCS(1) = SCS(1) XOR SCC(1)	TCC(1) = SCS(0) AND SCC(0)
10	TCS(0) = SCS(0) XOR SCC(0)	TCC(0) = '0'

TABLE 3

In the fourth level, a final SUM vector is calculated from the third level outputs **TCS(3)** to **TCS(0)** and **TCC(3)** to **TCC(0)** as shown in **Table 4**.

	Final_Sum
15	-----
	SUM(3) = TCS(3) XOR TCC(3)
	SUM(2) = TCS(2) XOR TCC(2)
	SUM(1) = TCS(1) XOR TCC(1)
	SUM(0) = TCS(0) XOR TCC(0)

TABLE 4

As mentioned, the Pre-correction Factor **PF(3)** to **PF(0)** is, in part, dependent upon the carries **C0** and **C4**. The Pre-correction Factor **PF(3)** to **PF(0)** is given below for each of four cases:

If the **C4** = '0' and **C0** = '0', the **PF(3)** to **PF(0)** is (TC3 0 0 0)-----Case1

If the **C4** = '0' and **C0** = '1', the **PF(3)** to **PF(0)** is (TC3 0 0 1)-----Case2

If the **C4** = '1' and **C0** = '0', the **PF(3)** to **PF(0)** is (TC3 1 1 0)-----Case3

If the **C4** = '1' and **C0** = '1', the **PF(3)** to **PF(0)** is (TC3 1 1 1)-----Case4

5 Case 1

The final SUM vector is obtained after substituting **PF(3)** to **PF(0)** values (TC3 0 0 0) in the First_Sum_Correction and First_Carry_Correction equations and then reducing the Second_Sum_Correction, Second_Carry_Correction, Third_Sum_Correction and Third_Carry_Correction equations. The finally reduced SUM vector is given below:

$$10 \quad \text{SUM}(3) = \text{TS}(3) \text{ XOR TC3} \quad \text{----- equ03}$$

$$\text{SUM}(2) = \text{TS}(2) \quad \text{----- equ02}$$

$$\text{SUM}(1) = \text{TS}(1) \quad \text{----- equ01}$$

$$\text{SUM}(0) = \text{TS}(0) \quad \text{----- equ00}$$

Case 2

15 The final SUM vector is obtained after substituting **PF(3)** to **PF(0)** values (TC3 0 0 1) in the First_Sum_Correction and First_Carry_Correction equations and then reducing the Second_Sum_Correction, Second_Carry_Correction, Third_Sum_Correction and Third_Carry_Correction equations. The finally reduced SUM vector is given below:

$$\text{SUM}(3) = \text{TS}(3) \text{ XOR TC3 XOR (TS}(0) \text{ AND TS}(1) \text{ AND TS}(2) \text{) } \text{-----equ13}$$

$$20 \quad \text{SUM}(2) = (\text{TS}(0) \text{ AND TS}(1)) \text{ XOR TS}(2) \quad \text{-----equ12}$$

$$\text{SUM}(1) = \text{TS}(0) \text{ XOR TS}(1) \quad \text{-----equ11}$$

$$\text{SUM}(0) = \text{NOT TS}(0) \quad \text{-----equ10}$$

Case 3

25 The final SUM vector is obtained after substituting **PF(3)** to **PF(0)** values (TC3 1 1 0) in the First_Sum_Correction and First_Carry_Correction equations and then reducing the

Second_Sum_Correction, Second_Carry_Correction, Third_Sum_Correction and Third_Carry_Correction equations. The finally reduced SUM vector is given below:

$$\text{SUM}(3) = \text{TS}(3) \text{ XOR } \text{TC3} \text{ XOR } (\text{TS}(1) \text{ OR } \text{TS}(2)) \quad \text{-----equ23}$$

$$\text{SUM}(2) = \text{TS}(1) \text{ XOR } (\text{NOT } \text{TS}(2)) \quad \text{-----equ22}$$

$$5 \quad \text{SUM}(1) = \text{NOT } \text{TS}(1) \quad \text{-----equ21}$$

$$\text{SUM}(0) = \text{TS}(0) \quad \text{-----equ20}$$

Case 4

The final SUM vector is obtained after substituting **PF(3)** to **PF(0)** values (TC3 1 1 1) in the First_Sum_Correction and First_Carry_Correction equations and then reducing the
10 Second_Sum_Correction, Second_Carry_Correction, Third_Sum_Correction and Third_Carry_Correction equations. The finally reduced SUM vector is given below

$$\text{SUM}(3) = \text{TS}(3) \text{ XOR } \text{TC}(3) \text{ XOR } (\text{TS}(0) \text{ OR } \text{TS}(1) \text{ OR } \text{TS}(2)) \quad \text{-----equ33}$$

$$\text{SUM}(2) = (\text{TS}(0) \text{ OR } \text{TS}(1)) \text{ XOR } (\text{NOT } \text{TS}(2)) \quad \text{-----equ32}$$

$$\text{SUM}(1) = \text{TS}(0) \text{ XOR } (\text{NOT } \text{TS}(1)) \quad \text{-----equ31}$$

$$15 \quad \text{SUM}(0) = \text{NOT } \text{TS}(0) \quad \text{-----equ30}$$

The implementation of **SUM(0)** correction circuit from equ00, equ10, equ20 and equ30 is shown in **FIG. 8**. The implementation of **SUM(1)** correction circuit from equ01, equ11, equ21 and equ31 is shown in **Fig. 9**. The implementation of **SUM(2)** correction circuit from equ02, equ12, equ22 and equ32 is shown in **Fig. 10**. The implementation of
20 **SUM(3)** correction circuit from equ03, equ13, equ23 and equ33 is shown in **Fig. 11**.

Clearly, there are equivalent circuits to those shown in **Figs. 8-11** for each of the other sum correction circuits **19, 20, 21**, to generate **SUM[7:4]**, **SUM[11:8]**, and **SUM[15:12]** values.

Final Sum

25 The Final Sum of the BCD addition is the sequence of **SUM[15:12]**, **SUM[11:8]**, **SUM[7:4]**, **SUM [3:0]**. A final carry, equal to **C16**, also results as an output.

Critical Paths

In the BCD addition circuit **12** as described, there are a number of critical paths in the three stages. Critical paths determine the number of logic levels required, and the speed at which addition can be performed.

5 Critical paths in the first stage:

CRITICAL_PATH_1 - The path through circuits **22, 23** and **24** which generates intermediate sum vector **TS(3)** to **TS(0)** from the BCD input operands **A(3)** to **A(0)** and **B(3)** to **B(0)** contains three XOR gates, and so the gate levels for this critical path is considered as 6.

- 10** CRITICAL_PATH_2a - The path through circuits **22, 23** and gates **26, 28, 30** from the BCD input operands to the generate function requires 7 gate levels.

CRITICAL_PATH_2b - The path through circuits **22, 23** and gates **25, 27, 30** from the BCD input operands to the generate function requires 7 gate levels.

- 15** CRITICAL_PATH_2c - The path through circuits **22, 23, 24** and gate **29** from the BCD input operands to the propagate function requires 7 gate levels.

Critical paths in the second stage:

CRITICAL_PATH_3 - The carry look ahead circuit for 16-bit BCD addition (circuit **17** in Fig. 3) requires two gate levels of logic.

Critical paths in the third stage:

- 20** CRITICAL_PATH_4 - The SUM correction circuit's critical path (gates **77, 78** and MUX **81** of Fig.11) from first stage output intermediate sum vector to final SUM vector requires 6 levels of gates, of which 4 levels are calculated before receiving the carries from the carry look ahead circuit **17**.

- 25** CRITICAL_PATH_5 - The path from the carry look ahead output carries to the final SUM which has 2 gate levels (MUX **62** or MUX **66** or MUX **73** or MUX **81**).

In the case of the 16-bit BCD adder circuit, the critical path is through SUM correction circuit and the path is CRITICAL_PATH_1 (6 gate levels) + CRITICAL_PATH_4 (6 gate levels). So the 16-bit BCD adder circuit has a delay of only 12 gate levels.

In the case of a 64-bit or 32 bits BCD adder circuit, the critical path is through the Carry look ahead circuit and the path is CRITICAL_PATH_2a or CRITICAL_PATH_2b or CRITICAL_PATH_2c (all are 7 gate levels) + CRITICAL_PATH_3 (6 gate levels) + CRITICAL_PATH_5 (2 gate levels). Therefore, 64-bit or 32-bit BCD adder circuits have
5 15 gate levels of logic in the critical path.

The 64-Bit adder circuit

The 16-bit BCD adder of **Fig. 2** can be extended to a 32-bit or 64-bit form with extra carry look ahead network levels.

A 64-bit BCD adder circuit includes three stages as discussed with reference to the 16-bit adder **12** shown in **Fig. 2**. As shown in **Fig. 12**, Stage 1 has sixteen logic circuits **90, 92, ..., 118, 120** of the type identified by numeral **13** in **Fig. 3**.
10

The circuits **90-120** receive contiguous 4-bit operands **A[3:0]** and **B[3:0]**, **A[7:4]** and **B[7:4]**, and so on, and generate respective intermediate sums **TS[3:0]**, ... **TS[63:60]**, intermediate carries **TC3, ..., TC63**, propagate functions **P3-0, ..., P63-60**, and generate
15 functions **G3-0, ..., G63-60**.

The second stage carry look ahead circuits are shown in **Fig. 13**. Five carry look ahead circuits **130, 132, 134, 136, 138** are required. These circuits are of the same for as the circuit **17** shown in **Fig. 7**.

The carry look ahead circuits are in two sub-stages: four circuits **130, 132, 134, 136**
20 coupled with the first stage circuits **90-120** to receive 4-bit propagate and generate functions, and a single circuit **138**, receiving 16-bit propagate and generate functions from the first sub-stage. The outputs from the second sub-stage to the third stage are the carries **C16, C32, C48, and C64**.

The third stage, shown in **Fig. 14** has sixteen sum correction circuits **150, 152, ..., 178, 180**. Each of these circuits is implemented in the same manner as, for example, sum correction circuit **18** described with reference to **Figs. 8-11**. Together, the sum correction circuits **150-180** calculate **SUM[3:0]** to **SUM [63:60]** giving the sum result of the operands **A[63:0] + B[63:0] + C₀**. The final carry is the value **C64**.
25

The 64-bit or 32-bit BCD adder requires six gate levels as shown in **Fig. 13** (circuits **130, 132, 134, 136** and **138**) in the second stage critical path.
30

The extension from 16-bit to 64-bit addition can be achieved with high reusability. A 64-bit adder circuit can be built by extending (i.e. reusing) the components of a 16-bit circuit four times, and adding only one extra level of a carry look ahead circuit.

Conclusion

- 5 The circuit described reduces the number of gate levels in the addition of two BCD-encoded operands with an input carry and makes the BCD addition fast.

While the invention contemplates equal length blocks for the operands, the embodiment taught is 4-bit blocks. For the 16-bit adder, the blocks are of contiguous bits from the
10 least to most significant bits. The use of 4-bit blocks is the most preferred, however, in that the fan-in and fan-out of carry look ahead logic is the least onerous to implement in hardware.

Various alterations and modifications can be made to the techniques and arrangements
15 described herein, as would be apparent to one skilled in the relevant art.